

Appendix A

Grade 11 U/C Mathematics
Scope and Sequence

Unit	Total number of lessons	Lessons Included	Instructional Jazz Days/ Review Days	Summative Evaluation Days	Total Number of Days
1. Introductory Unit	4	4	0	0	4
2. Functions Through Quadratics (Broad Strokes)	6	3	0	1	7
3. Investigating Quadratics	9	4	1	1	11
4. Quadratic Highs and Lows	13	8	2	3*	18
5. Exponential Functions	10	8	1	1	12
6. Financial Applications of Exponential Functions	8	3	1	1	10
7. Acute Triangle Trigonometry	5	3	1	1	7
8. Trigonometric Functions	9	5	1	1	11
Course Review	2	0	0	0	2
Course Summative Performance Task	0	0	0	2	2
Totals	66	38	7	11	84





* The midterm summative assessment is a two day performance task that covers units 2 through 4


Note: Days are defined to have 76 minutes.

Unit # 1: Introduction (4 days + 0 jazz days + 0 summative evaluation days)

BIG Ideas:

- This is an opportunity for students to see the big picture of the course.
- Students will explore 4 functions (linear, quadratic, exponential and periodic) in a very general way.
- Having students “walk” each of these graphs will give them a kinesthetic connection with the similarities and differences


DAY	Lesson Title & Description	2P	2D	Expectations	Teaching/Assessment Notes and Curriculum Sample Problems	
1	Walk the Line <ul style="list-style-type: none"> • Review of DT graphs <p><i>Lesson Included</i></p>			Review of Grade 10		
2	Lines, Curves and Waves Oh My! <ul style="list-style-type: none"> • Investigate Linear, Quadratic, Exponential and Periodic Graphs with the CBR <p><i>Lesson Included</i></p>	N	N	EF1.05 ✓	distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth)	<p><i>Sample problem:</i> Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$.</p> <div style="text-align: right;"></div>
		N	N	TF3.01 <input checked="" type="checkbox"/>	<u>collect data</u> that can be modelled as a sine function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g. websites such as Statistics Canada, E-STAT), and graph the data	<p><i>Sample problem:</i> Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.</p> <div style="text-align: right;"></div>
3	A Cube Conundrum <ul style="list-style-type: none"> • Compare and contrast linear and quadratic relationships • Students begin to group relationships into categories of linear, quadratic or other <p><i>Lesson Included</i></p>			EF1.05 ✓ TF3.01 <input checked="" type="checkbox"/>		


4	<u>Getting Ready for the Journey</u> <ul style="list-style-type: none"> Classify relationships as linear, quadratic or other Explore some characteristics of exponential and periodic relationships 			EF1.05 ✓		
				TF3.01 ☑		
<i>Lesson Included</i>						

Unit 2: Functions (6 days + 0 jazz days + 1 summative evaluation day)

BIG Ideas:

- quadratic expressions can be expanded and simplified
- the solutions to quadratic equations have real-life connections
- properties of quadratic functions
- problems can be solved by modeling quadratic functions



DAY	Lesson Title & Description	2P	2D	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	<u>Is It or Isn't It?</u> <ul style="list-style-type: none"> Explore relations in various forms to determine it is a function A vertical line test can be used to determine if a graph is a function <i>Lesson Included</i>	N	N	QF2.01 <input checked="" type="checkbox"/>	explain the meaning of the term <i>function</i> , and <u>distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations</u> (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., using the vertical line test)	Sample problem: Investigate, using numeric and graphical representations, whether the relation $x = y^2$ is a function, and justify your reasoning.);
2	<u>Frame It</u> <ul style="list-style-type: none"> Students will investigate and model quadratic data <i>Lesson Included</i>	C	C	QF3.01 ✓	collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	Sample problem: When a $3 \times 3 \times 3$ cube made up of $1 \times 1 \times 1$ cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function. 
3	<u>Applications of Linear & Quadratic Functions</u> <ul style="list-style-type: none"> Evaluate functions using function notation 	N	N	QF2.02 <input checked="" type="checkbox"/>	substitute into and <u>evaluate linear and quadratic functions</u> represented using function notation [e.g., evaluate $f(\frac{1}{2})$, given $f(x) = 2x^2 + 3x - 1$], including functions arising from real-world applications	Sample problem: The relationship between the selling price of a sleeping bag, s dollars, and the revenue at that selling price, $r(s)$ dollars, is represented by the function $r(s) = -10s^2 + 1500s$. Evaluate, interpret, and compare $r(29.95)$, $r(60.00)$, $r(75.00)$, $r(90.00)$, and $r(130.00)$.


4	<u>Home on the Range</u> <ul style="list-style-type: none"> Different notations of domain and range of functions in various forms will be explored <p><i>Lesson Included</i></p>	N	N	QF2.03 QF2.04 <input checked="" type="checkbox"/>	explain the meanings of the terms <i>domain</i> and <i>range</i> , through investigation using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately (e.g., for $y = x^2 + 1$, the domain is the set of real numbers, and the range is $y \geq 1$); explain any restrictions on the domain and the range of a quadratic function in contexts arising from real-world applications	<p>Sample problem: A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?</p> 
5	<u>Applications of Quadratic Relations Part I</u> <ul style="list-style-type: none"> Create and solve real-world problems using tables and graphs 	C	C	QF1.01 <input checked="" type="checkbox"/>	pose and solve problems involving quadratic relations arising from real-world applications and represented by tables of values and graphs (e.g., "From the graph of the height of a ball versus time, can you tell me how high the ball was thrown and the time when it hit the ground?");	
6	<u>Applications of Quadratic Relations Part II</u> <ul style="list-style-type: none"> Solve real-world problems using an algebraic representation of a quadratic function. 	C	C	QF3.03 <input checked="" type="checkbox"/>	solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement)	<p>Sample problem: In a DC electrical circuit, the relationship between the power used by a device, P (in watts, W), the electric potential difference (voltage), V (in volts, V), the current, I (in amperes, A), and the resistance, R (in ohms, Ω), is represented by the formula $P = IV - I^2 R$. Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 Ω. Determine the current needed in order for the device to use the maximum amount of power.</p>
7	<u>Summative Unit Evaluation</u>					

Unit #3: Investigating Quadratics (9 days + 1 jazz day + 1 summative evaluation day)

BIG Ideas:

- Developing strategies for determining the zeroes of quadratic functions
- Making connections between the meaning of zeros in context
- quadratic data can be modeled using algebraic techniques



DAY	Lesson Title & Description	2P	2D	Expectations	Teaching/Assessment Notes and Curriculum Sample Problems
1	<p><u>The Zero Connection</u></p> <ul style="list-style-type: none"> • students explore the connections between the x-intercepts and the roots of a quadratic equation <p><i>Lesson Included</i></p>	C	R	<p>QF1.05 ✓</p> <p>determine, through investigation, and describe the connection between the factors used in solving a quadratic equation and the x-intercepts of the corresponding quadratic relation</p>	<p>Sample problem: The profit, P, of a video company, in thousands of dollars, is given by $P = -5x^2 + 550x - 5000$, where x is the amount spent on advertising, in thousands of dollars. Determine, by factoring and by graphing, the amount spent on advertising that will result in a profit of \$0. Describe the connection between the two strategies.</p> 
2	<p><u>The simple Life</u></p> <ul style="list-style-type: none"> • students explore different representations for expanding and simplifying quadratic expressions <p><i>Lesson Included</i></p>	C	C	<p>QF1.02 ✓</p> <p>represent situations (e.g., the area of a picture frame of variable width) using quadratic expressions in one variable, and expand and simplify quadratic expressions in one variable [e.g., $2x(x + 4) - (x+3)^2$];*</p>	<p><i>*The knowledge and skills described in this expectation may initially require the use of a variety of learning tools (e.g., computer algebra systems, algebra tiles, grid paper.</i></p> 
3,4	<p><u>Factoring Quadratics</u></p> <ul style="list-style-type: none"> • Factor both simple and complex trinomials • Factor, through exploration, different types of trinomials 	Have only done simple trinomials	C	<p>QF1.03 ✓</p> <p>factor quadratic expressions in one variable, including those for which $a \neq 1$ (e.g., $3x^2 + 13x - 10$), differences of squares (e.g., $4x^2 - 25$), and perfect square trinomials (e.g., $9x^2 + 24x + 16$), by selecting and applying an appropriate strategy (</p>	<p>Sample problem: Factor $2x^2 - 12x + 10$.);</p> <p><i>The knowledge and skills described in this expectation may initially require the use of a variety of learning tools (e.g., computer algebra systems, algebra tiles, grid paper.</i></p>






5,6	<u>Solving Quadratic Equations</u> <ul style="list-style-type: none"> Solve equations by factoring 	N	R	QF1.04 ✓	solve quadratic equations by selecting and applying a factoring strategy;	
7	<u>Back to the Future</u> <ul style="list-style-type: none"> connect graphic, algebraic and written representations determine solutions to problems in context involving applications of quadratic functions <p><i>Lesson Included</i></p>	C	C	QF3.03 ✓	solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement) [<p>Sample problem: In a DC electrical circuit, the relationship between the power used by a device, P (in watts, W), the electric potential difference (voltage), V (in volts, V), the current, I (in amperes, A), and the resistance, R (in ohms, Ω), is represented by the formula $P = IV - I^2 R$. Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 Ω. Determine the current needed in order for the device to use the maximum amount of power.]</p>
8	<u>Exploring Quadratic Phenomena</u> <ul style="list-style-type: none"> Collect and explore quadratic data either through experiment or using pre-made data 	C	C	QF3.01 ✓	collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data;	<p>Sample problem: When a $3 \times 3 \times 3$ cube made up of $1 \times 1 \times 1$ cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.</p>
9	<u>Math's Next Top Model</u> <ul style="list-style-type: none"> students will determine an appropriate quadratic model using x-intercepts and one point <p><i>Lesson Included</i></p>	N	N	QF3.02 ✓	determine, through investigation using a variety of strategies (e.g., applying properties of quadratic functions such as the x-intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology (e.g., graphing software, graphing calculator);	<p><i>Suggestion....Bouncing ball...students can find the equation using the x-intercepts and 1 point</i></p> 
10	<u>Review Day (Jazz Day)</u>					
11	<u>Summative Unit Evaluation</u>					


Unit #4: Quadratic - Highs and Lows (13 days + 2 jazz + 3 midterm summative evaluation days)




BIG Ideas:

- Investigate the three forms of the quadratic function and the information that each form provides.
- Using technology, show that all three forms for a given quadratic function are equivalent.
- Convert from standard (expanded) form to vertex form by completing the square.
- Sketch the graph of a quadratic function by using a suitable strategy. (i.e. factoring, completing the square and applying transformations)
- Explore the development of the quadratic formula and connect the value of the discriminant to the number of roots.
- Collect data from primary and secondary sources that can be modelled as a quadratic function using a variety of tools.
- Solve problems arising from real world applications given the algebraic representation of the quadratic function.

DAY	Lesson Title & Description	2P	2D	Expectations	Teaching/Assessment Notes and Curriculum Sample Problems
1	<p>Graphs of quadratics in factored Form</p> <ul style="list-style-type: none"> • The zeros and one other point are necessary to have a unique quadratic function • Determine the coordinates of the vertex from your sketch or algebraic model <p><i>Lesson Included</i></p>	C No "a"	C	<p>QF2.09 ✓</p> <p>sketch graphs of quadratic functions in the factored form $f(x) = a(x - r)(x - s)$ by using the x- intercepts to determine the vertex;</p>	 Computer and data projector (Optional)
2	<p>Investigating the roles of a, h and k in the Vertex Form</p> <ul style="list-style-type: none"> • Investigate the roles of "a", "h" and "k" • Apply a series of transformation to $y=x^2$ to produce the necessary quadratic function <p><i>Lesson Included</i></p>	N	C	<p>QF2.05 ✓</p> <p>determine, through investigation using technology, and describe the roles of a, h, and k in quadratic functions of the form $f(x) = a(x - h)^2 + k$ in terms of transformations on the graph of $f(x) = x^2$ (i.e., translations; reflections in the x-axis; vertical stretches and compressions)</p>	 Computer Lab <p>Sample problem: Investigate the graph $f(x) = 3(x - h)^2 + 5$ for various values of h, using technology, and describe the effects of changing h in terms of a transformation.</p>

3	<p><u>Sketching quadratics functions in vertex form</u></p> <ul style="list-style-type: none"> Apply a series of transformation to $y=x^2$ to produce the necessary quadratic function <p><i>Lesson Included</i></p>	N	C	QF2.06 ✓	sketch graphs of $g(x) = a(x - h)^2 + k$ by applying one or more transformations to the graph of $f(x) = x^2$	 Computer and data projector <p>Sample problem: Transform the graph of $f(x) = x^2$ to sketch the graphs of $g(x) = x^2 - 4$ and $h(x) = -2(x + 1)^2$</p>
4	<p><u>Changing from vertex form to standard (expanded) form</u></p> <ul style="list-style-type: none"> Verify using technology that both forms are equivalent 	N	N	QF2.07 ✓	express the equation of a quadratic function in the standard form $f(x) = ax^2 + bx + c$, given the vertex form $f(x) = a(x - h)^2 + k$, and verify, using graphing technology, that these forms are equivalent representations	 or  <p>Sample problem: Given the vertex form $f(x) = 3(x - 1)^2 + 4$, express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.</p>
5, 6	<p><u>Completing the Square</u></p> <ul style="list-style-type: none"> Use algebra tiles to investigate procedures Verify using technology that both forms are equivalent Develop a procedure to complete the square using algebra <p><i>Lessons Included</i></p>	N	C	QF2.08 ☑	express the equation of a quadratic function in the vertex form $f(x) = a(x - h)^2 + k$, given the standard form $f(x) = ax^2 + bx + c$ by completing the square (e.g., using algebra tiles or diagrams; algebraically), including cases where $\frac{b}{a}$ is a simple rational number (e.g., $\frac{1}{2}$, 0.75), and verify, using graphing technology, that these forms are equivalent representations;	 Algebra Tiles Day 5  Day 6
7	<p><u>Gathering information from the three forms of quadratic functions</u></p> <ul style="list-style-type: none"> Use inspection to gather information 	N	N	QF2.10 ✓	describe the information (e.g., maximum, intercepts) that can be obtained by inspecting the standard form $f(x) = ax^2 + bx + c$, the vertex form $f(x) = a(x - h)^2 + k$, and the factored form $f(x) = a(x - r)(x - s)$ of a quadratic function;	
8	<p><u>Sketching the graph of quadratic functions in standard form</u></p> <ul style="list-style-type: none"> Use a suitable strategy to gather information to construct the graph 	N	R	QF2.11 ✓	sketch the graph of a quadratic function whose equation is given in the standard form $f(x) = ax^2 + bx + c$ by using a suitable strategy (e.g., completing the square and finding the vertex; factoring, if possible, to locate the x -intercepts), and identify the key features of the graph (e.g., the vertex, the x - and y -intercepts, the equation of the axis of symmetry, the intervals where the function is positive or negative, the intervals where the function is increasing or decreasing).	

9	<p><u>"CAS"ing out the quadratic formula</u></p> <ul style="list-style-type: none"> Explore the development of the quadratic formula using CAS Apply the formula to solve equations using technology <p><i>Lesson Included</i></p>	N	R	QF1.06 ✓	explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development, with technology, such as computer algebra systems, or without technology [student reproduction of the development of the general case is not required]), and apply the formula to solve quadratic equations, using technology;	 CAS
10	<p><u>Relating roots and zeros of quadratic functions</u></p> <ul style="list-style-type: none"> X-intercepts (zeros) and roots are synonymous The sign of the discriminant determines the number of roots 	N	N	QF1.07 ✓	relate the real roots of a quadratic equation to the x -intercept(s) of the corresponding graph, and connect the number of real roots to the value of the discriminant (e.g., there are no real roots and no x -intercepts if $b^2 - 4ac < 0$);	
11	<p><u>Solving quadratic equations</u></p> <ul style="list-style-type: none"> Solve equations using a variety of strategies Describe advantages and disadvantages of each strategy 	N	C	QF1.08 ✓	determine the real roots of a variety of quadratic equations (e.g., $100x^2 = 115x + 35$), and describe the advantages and disadvantages of each strategy (i.e., graphing; factoring; using the quadratic formula)	<p>Sample problem: Generate 10 quadratic equations by randomly selecting integer values for a, b, and c in $ax^2 + bx + c = 0$. Solve the equations using the quadratic formula. How many of the equations could you solve by factoring?).</p>


12, 13	Nano Project or Fuel Fit <ul style="list-style-type: none"> Collect data from primary or secondary sources without technology Determine the equation of a quadratic model for the collected data using technology Solve problems from real world applications given the algebraic representation of a quadratic function <p><i>Lessons Included</i></p>	C	C	QF3.01 ✓	collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	<p>Sample problem: When a $3 \times 3 \times 3$ cube made up of $1 \times 1 \times 1$ cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.)</p> <p>Sample problem: When a $3 \times 3 \times 3$ cube made up of $1 \times 1 \times 1$ cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.)</p>  or 
		N	N	QF3.02 ✓	determine, through investigation using a variety of strategies (e.g., applying properties of quadratic functions such as the x -intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology (e.g., graphing software, graphing calculator);	
		C	C	QF3.03 ✓	solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement)	
14	Instructional jazz day Note: This day may be located throughout the unit as needed.					
15, 16	Midterm summative assessment performance task Note: Two possible performance tasks are included (A Leaky Problem or Bridging the Gap)					 (Day 16) Note: The two midterm summative performance tasks are in the file Midterm SP Task .
17	Unit Review					
18	Pencil and paper summative assessment on expectations from this unit not covered in the summative performance task.					


Unit #5 : Exponential Functions (10 days + 1 jazz day + 1 summative evaluation day)



BIG Ideas:



Students will:


- Collect primary data and investigate secondary data that can be modelled as exponential growth/decay functions
- Make connections between numeric, graphical and algebraic representations of exponential functions
- Identify key features of the graphs of exponential functions (e.g., domain, range, y-intercept, horizontal asymptote, increasing and decreasing)
- Apply an understanding of domain and range to a variety of exponential models
- Solve real-world applications using given graphs or equations of exponential functions
- Simplify and evaluate numerical expressions involving exponents

DAY	Lesson Title & Description	2P	2D	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	<p>Piles of Homework</p> <ul style="list-style-type: none"> • Distinguish exponential functions from linear and quadratic by examining tables of values and graphs <p><i>Lesson Included</i></p>	N	N	EF1.06 ✓	distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth)	<p>Sample problem: Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$.</p>
2	<p>Investigating Exponential Growth</p> <ul style="list-style-type: none"> • Collect data that can be modelled as exponential growth functions through investigation and from secondary sources • Make connections to First Differences and constant ratios <p><i>Lesson Included</i></p>	N	N	EF2.01 ✓	collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	<p>Sample problem: Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.)</p> 

3	<u>Investigating Exponential Decay</u> <ul style="list-style-type: none"> Collect data that can be modelled as exponential decay functions Make connections to First Differences and constant ratios <p><i>Lesson Included</i></p>	N	N	EF2.01 ✓		
4	<u>Investigating The Graphs of Exponential Functions – Day 1</u> <ul style="list-style-type: none"> Graph exponential functions in the form $y=ab^x$ where $b>0$ and $a=1$ Identify key features (y-intercept, increasing or decreasing, domain and range, horizontal asymptotes, constant ratio) <p><i>Lesson Included</i></p>	N	N	EF1.03 ✓	graph, with and without technology, an exponential relation, given its equation in the form $y = a^x$ ($a > 0, a \neq 1$), define this relation as the function $f(x) = a^x$, and explain why it is a function;	<p>Sample problem: Graph $f(x) = 2^x$, $g(x) = 3^x$, and $h(x) = 0.5^x$ on the same set of axes. Make comparisons between the graphs, and explain the relationship between the y-intercepts.</p>
5	<u>Investigating The Graphs of Exponential Functions – Day 2</u> <ul style="list-style-type: none"> Graph exponential functions in the form $y=ab^x$ where $b>0$ and $a>1$ Identify key features (y-intercept, increasing or decreasing, domain and range, horizontal asymptotes, constant ratio) <p><i>Lesson Included</i></p>	N	N	EF1.04 ✓	determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x) = a^x$ ($a > 0, a \neq 1$), function machines]	

6	<p><u>Domain and Range in Real World Applications</u></p> <ul style="list-style-type: none"> Identify exponential functions that arise from real world applications involving growth and decay Determine reasonable restrictions on the domain and range <p><i>Lesson Included</i></p>	N	N	EF2.02 ✓	<p>identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve);</p>	
7	<p><u>How an Infectious Disease can Spread</u></p> <ul style="list-style-type: none"> Simulate the spread of an infectious disease and analyze the results Determine restrictions that must be placed on the domain and range in order to apply an exponential model <p><i>Lesson Included</i></p>	N	N	EF2.02 ✓	<p>identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve);</p>	<p>This activity requires advanced preparation. Full Teacher Notes are provided in BLM 5.7.2</p> 


8,9	<p><u>Developing and Applying Exponent Laws</u></p> <ul style="list-style-type: none"> Investigate to develop exponent laws for multiplying and dividing numerical expressions involving exponents and for finding the power of a power. Investigate to find the value of a power with a rational exponent (e.g., use a graphing calculator to find the value for $4^{\frac{1}{2}}$ or $27^{\frac{1}{3}}$ by entering an exponential function with the given base and then using TRACE.) Evaluate numerical expressions with rational bases and integer/rational exponents. Note: Students only work with numerical expressions 	N	C	EF1.05 ✓	determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numerical expressions involving exponents [e.g., $(\frac{1}{2})^3 \times (\frac{1}{2})^2$], and the exponent rule for simplifying numerical expressions involving a power of a power [e.g., $(5^3)^2$], and use the rules to simplify numerical expressions containing integer exponents [e.g., $(2^3)(2^5) = 2^8$];	<p>Note: Students don't actually solve exponential equations in this course so the main use of these exponent rules would likely be to help develop an understanding of rational exponents (see sample problem below) and to understand the compound interest formula</p> <hr/> <p>Sample problem: The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$. What value would you assign to $4^{\frac{1}{2}}$? What value would you assign to $27^{\frac{1}{3}}$? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$, where $x > 0$ and n is a natural number.</p> <p>Suggestion: Teachers may want to have students explore on sketchpad or with a graphing calculator. Students can graph $y = 4^x$ and then examine the y-value when $x = \frac{1}{2}$ and then graph $y = 9^x$ and examine the y-value when $x = \frac{1}{2}$ and so on.</p> <p> and/or </p>
		N	N	EF1.01 ☑	<u>determine, through investigation using a variety of tools</u> (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$, where $x > 0$ and m and n are integers)	
		N	C	EF1.02 ☑	<u>evaluate, with and without technology,</u> numerical expressions containing integer and rational exponents and rational bases [e.g., 2^{-3} , $(-6)^3$, $4^{\frac{1}{2}}$, 1.01^{120}];	





10	<p><u>Using Graphical and Algebraic Models</u></p> <ul style="list-style-type: none"> • Students will solve problems using given graphs or equations of exponential functions • Help students make connections between the algebraic model of the exponential function and the real-world application (i.e. help students understand the meanings of a and b in the context of the problem) • Note: Students are not required to generate the equation on their own, but should be encouraged to explain the parameters in the context of the problem. <p><i>Lesson Included</i></p>	N	N	EF2.03 ✓	<p>solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations</p>	<p>Sample problem: The temperature of a cooling liquid over time can be modelled by the exponential function</p> $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$ <p>where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28°C.</p> 
11	<p><u>Review Day (Jazz Day)</u></p>					
12	<p><u>Summative Unit Evaluation</u></p>					


Unit 6: Financial Applications of Exponential Functions (8 days + 1 jazz day + 1 summative evaluation day)

BIG Ideas:

- Connecting compound interest to exponential growth
- Examining annuities using technology
- Making decisions and comparisons using the TVM solver

DAY	Lesson Title & Description	2P	2D	Expectations	Teaching/Assessment Notes and Curriculum Sample Problems	
1	<u>Interested in Your Money</u> <ul style="list-style-type: none"> • Investigating and defining financial terminology • Calculating and comparing simple and compound interest <p><i>Lesson Included</i></p>	N	N	EF3.01 ✓	compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time	 <p>Sample problem: Compare, using tables of values and graphs, the amounts after each of the first five years for a \$1000 investment at 5% simple interest per annum and a \$1000 investment at 5% interest per annum, compounded annually.</p>
2	<u>Connecting Compound Interest & Exponential Growth</u> <ul style="list-style-type: none"> • Connecting simple interest with linear growth • Connecting compound interest with exponential growth <p><i>Lesson Included</i></p>	N	N	EF3.01 ✓	determine, through investigation (e.g., using spreadsheets and graphs), that compound interest is an example of exponential growth [e.g., the formulas for compound interest, $A = P(1 + i)^n$, and present value, $PV = A(1 + i)^{-n}$, are exponential functions, where the number of compounding periods, n , varies]	<p>Sample problem: Describe an investment that could be represented by the function $f(x) = 500(1.01)^x$.</p>
		N	N	EF3.03 ✓		
3	<u>There's Gotta Be a Faster Way</u> <ul style="list-style-type: none"> • Developing the compound interest formula • Solving problems using the compound interest formula 	N	N	EF3.03 ✓	solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), and the principal, P (also referred to as present value, PV), using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$]	<p><i>Use homework from Day 2 as a rationale for finding a formula to calculate compound interest. Make connections between constant ratio in the table from this homework and the $(1+i)$ in the formula.</i></p>
		N	N	EF3.02		<p>Sample problem: Calculate the amount if \$1000 is invested for three years at 6% per annum, compounded quarterly.</p>

4	<u>TVM Solver</u> <ul style="list-style-type: none"> • Introduction on how to use the TVM solver for compound interest • Using TVM solver to calculate time and interest rates 	N	N	EF3.04	solve problems, using a TVM Solver in a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period, i , or the number of compounding periods, n , in the compound interest formula $A = P(1 + i)^n$ [or $FV = PV(1+i)^n$]	 <i>Note: To introduce the TVM solver to students, use the TVM solver to check answers to previous day's work.</i> Sample problem: Use the TVM Solver in a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.
5	<u>Annuities</u> <ul style="list-style-type: none"> • Defining the term annuity • Investigate the amount of an annuity using technology 	N	N	EF3.05 ✓	explain the meaning of the term <i>annuity</i> , through investigation of numerical and graphical representations using technology;	 or
		N	N	EF3.07 ☑	solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary annuity <u>in situations where the compounding period and the payment period are the same</u> (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).	<i>Note: it may be helpful to explore annuities on a spreadsheet first so that students can “see” how the investment is growing.</i> <i>Note: it is not an expectation to develop or use an annuity formula. Students should always use a spreadsheet or a TVM solver.</i>
6	<u>Saving</u> <ul style="list-style-type: none"> • Exploration of annuities involving earning interest • Examining total interest 	N	N	EF3.07 ☑		 or
7	<u>Borrowing</u> <ul style="list-style-type: none"> • Exploration of annuities involving paying interest (loans) • Examining total interest 					 or



8	<p><u>Changing Conditions</u></p> <ul style="list-style-type: none"> Examine annuity scenarios where conditions are changed (payments, interest rate, etc) and make conclusions about the effects. <p><i>Lesson Included</i></p>	N	N	EF3.06 <input checked="" type="checkbox"/>	<p>determine, through investigation using technology (e.g., the TVM Solver in a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary annuities <u>in situations where the compounding period and the payment period are the same</u> (e.g., long-term savings plans, loans)</p>	 <p>Sample problem: Compare the amounts at age 65 that would result from making an annual deposit of \$1000 starting at age 20, or from making an annual deposit of \$3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?);</p>
9	<p><u>Review Day (Jazz Day)</u></p>					
10	<p><u>Summative Unit Evaluation</u></p>					

Unit 7: Acute Triangle Trigonometry (5 days + 1 jazz day + 1 summative evaluation day)

BIG Ideas:

Students will:

- Solve acute triangles using the primary trigonometric ratios, sine law, and cosine law
- Solve real-world application problems requiring the use of the primary trigonometric ratios, sine law, and cosine law including 2-D problems involving 2 right triangles

DAY	Lesson Title & Description	2P	2D	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	Remember SOHCAHTOA? <ul style="list-style-type: none"> • Solve right angled triangle problems using SOHCAHTOA • Solve questions involving 2 right triangles (NO 3-D triangles) 	R N	R N	TF1.01 ✓	solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios;	
2*	Investigating Sine law <ul style="list-style-type: none"> • Investigate Sine Law using GSP • Solve problems involving Sine Law <p><i>Lesson Included</i></p>	N	R	TF1.03 ✓	verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$, and in triangle ABC while dragging one of the vertices);	 (with GSP)
3*	Investigating Cosine Law <ul style="list-style-type: none"> • Investigate Cosine Law using GSP • Solve problems involving Cosine law • Discuss when to use Sine Law vs. Cosine Law vs. SOHCAHTOA <p><i>Lesson Included</i></p>	N N	R N	TF1.03 ✓ TF1.04 ✓	describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;	 (with GSP)

4,5	<u>Solving Problems Involving Sine and Cosine Law</u> <ul style="list-style-type: none"> • Tie up loose ends from days 2 & 3. • Discuss when to use Sine Law vs. Cosine Law vs. SOHCAHTOA • Do applications using sine and cosine law • Questions involving 2 right triangles (NO 3-D triangles) • Emphasize choosing appropriate tools for the question. <p><i>Activity Included</i></p>	N	N	TF1.04 ✓	describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;	
		N	N	TF1.05 ☑	solve problems that require <u>the use of the sine law or the cosine law in acute triangles</u> , including problems arising from real-world applications (e.g., surveying; navigation; building construction).	
		N	N	TF1.02	solve problems involving two right triangles in <u>two dimensions</u>	
6	<u>Review Day (Jazz Day)</u>					
7	<u>Summative Unit Evaluation</u>					





NOTE: * Depending on Technology access, you may wish to do both investigations (Day 2 and 3) on one day and then applications on the next day.





Unit 8: Trigonometric Functions (9 days + 1 jazz day + 1 summative evaluation day)


BIG Ideas:

Students will:




- Investigate periodic functions with and without technology.
- Study of the properties of periodic functions
- Study of the transformations of the graph of the sine function
- Solve real-world applications using sinusoidal data, graphs or equations

DAY	Lesson Title & Description	2P	2D	Expectations	Teaching/Assessment Notes and Curriculum Sample Problems	
1,2	Investigating Periodic Behaviour <ul style="list-style-type: none"> • Complete investigations to collect data • Follow-up with questions regarding cycle, amplitude and period, etc – without formally identifying them as such. <p><i>Lesson Included</i></p>	N	N	TF3.01 ✓	collect data that can be modelled as a sine function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	 + CBL 
		N	N	TF2.01	describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numerical or graphical representation;	
3	Introduction to Periodic Terminology <ul style="list-style-type: none"> • Discuss definitions of cycle, period, amplitude, axis of the curve, domain and range 	N	N	TF2.02 ✓	predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural-gas consumption in Ontario from previous consumption);	
4	Back and Forth and Round and Round <ul style="list-style-type: none"> • Investigation to discover the effect variations have on the graph of a periodic function <p><i>Lesson Included</i></p>	N	N	TF2.05 ✓	make connections, through investigation with technology, between changes in a real-world situation that can be modelled using a periodic function and transformations of the corresponding graph (e.g., investigating the connection between variables for a swimmer swimming lengths of a pool and transformations of the graph of distance from the starting point versus time)	 + CBL 

5	<u>Introduction to the Sine Function</u> <ul style="list-style-type: none"> Student led investigation to discover the Sine Function <p><i>Lesson Included</i></p>	N	N	TF2.03 ✓	make connections between the sine ratio and the sine function by graphing the relationship between angles from 0° to 360° and the corresponding sine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$, and explaining why it is a function;	<p><i>Note: these students will not have seen trig ratios with angles greater than 90°</i></p> 
		N	N	TF2.04 ☑	sketch the graph of $f(x) = \sin x$ for angle measures expressed in <u>degrees</u> , and determine and describe its key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals);	
6	<u>Discovering Sinusoidal Transformations</u> <ul style="list-style-type: none"> Using Graphing Calculator, discover the effects of a, c and d on the graph of $y = \sin x$ 	N	N	TF2.06	determine, through investigation using technology, and describe the roles of the parameters a , c , and d in functions in the form $f(x) = a \sin x$, $f(x) = \sin x + c$, and $f(x) = \sin(x - d)$ in terms of transformations on the graph of $f(x) = \sin x$ with angles expressed in degrees (i.e., translations; reflections in the x -axis; vertical stretches and compressions);	
7	<u>Graphing Sine Functions</u> <ul style="list-style-type: none"> Sketch the graphs of a given transformed sine function <p><i>Pair share wrap-up activity included BLM8.7.1</i></p>	N	N	TF2.07 ☑	sketch graphs of $f(x) = a \sin x$, $f(x) = \sin x + c$, and $f(x) = \sin(x - d)$ by applying transformations to the graph of $f(x) = \sin x$, <u>and state the domain and range of the transformed functions</u> (note: only 1 transformation at a time)	<p>Sample problem: Transform the graph of $f(x) = \sin x$ to sketch the graphs of $g(x) = -2\sin x$ and $h(x) = \sin(x - 180^\circ)$, and state the domain and range of each function</p>
8	<u>Applications of Sinusoidal Functions</u> <ul style="list-style-type: none"> Work on application problems Given a sine function graph the function using technology Use the graph to answer questions. <p><i>Lesson Included</i></p>	N	N	TF3.03 ☑	pose and solve problems based on applications involving a sine function by using a <u>given graph or a graph generated with technology from its equation</u>	  with projector & power point (not necessary)
				TF3.02 ✓	identify sine functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range;	
				TF2.02 ✓		

9	<u>What Goes Up Must Come Down</u> <ul style="list-style-type: none"> Graph sinusoidal data and find the curve of best fit (using TI-83s calculators) Use the graph to answer questions about the graph <p><i>Lesson Included</i></p>	N	N	TF3.02 ✓ TF3.03 ☑ TF2.02 ✓		 with projector & power point (not necessary)
10	<u>Review Day (Jazz Day)</u>					
11	<u>Summative Unit Evaluation</u>					

Key for symbols used in the outline:

- N** means a new concept for students coming into this course.
- C** means a continuing concept for students coming from this course.
- R** means this concept has been covered for students coming from this course so only review will be required.
- ✓ means a new expectation
- ☑ means the expectation has been revised slightly from 2000 curriculum.
-  means the lesson requires graphing calculators.
-  means the lesson requires a computer lab or at least one computer with projector for a teacher led investigation.
-  means the lesson requires manipulatives.